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ALGEBRAIC SIMPLIFICATION

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Abstract: Herein described are proposed and suggested changes and additions to Steve Russell's work on Simplify.

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Basic additions to the task of simplification routine to date attempt to extend the power of the program in several areas where modifications were straightforward and did not involve changes in the basic philosophy of simplification as incorporated by Steve Russell. For example, simplify now recognizes that an expression of the form $(\text{TIMES} (\text{MINUS } A) A)$ is equivalent to the form $(\text{MINUS} (\text{EXPT } A 2))$, which is preferred because it contains one less occurrence of A and because the minus sign has been brought to the outside of the expression. Perhaps a more significant change is that simplify can now collect exponents even when they are not all numeric; for example, $(\text{QUOTIENT} (\text{EXPT } X M) (\text{EXPT } X N))$ becomes $(\text{EXPT } X (\text{PLUS} (\text{MINUS } N) M))$. Other minor corrections have also been made to the simplify program.

Among the goals discussed earlier this month were the recognition of identities (in particular, trigonometric identities), and the facility to factor specified variables. It should be noted that there is an analogy between collecting exponents within a product and collecting like factors from terms of a sum. The difficulty in collecting factors lies in the commutative property of multiplication (as opposed to exponentiation) and the unknown number of factors within the product. Relatively few changes to the basic program were required to overcome these difficulties. This factorization leads to a form of expression which is similar to the form used by the polynomial simplify program described in an earlier memo of 5 June. Addition of the complete polynomial package is expected to be accomplished this summer.

Previously unused portions of the simplify program have been expanded to make possible efficient detection of instances of special forms, which are stored on the property list of the identity-recognizing function. For example, if the form $f(x) + g(x) = h(x)$ occurs on the identity list, then the expression $2g(y) + c(z) + 3f(y)$ will be simplified to $2h(y) + f(y) + c(z)$.

Remaining goals for the summer include the ability to expand expressions when desired and, in general, to orient the program toward easy man-machine interaction: in particular, a routine to translate Algol-like input expressions into Lisp S-expressions, expansion of macroprint's abilities, and mechanism to save intermediate results while working at the PDP-1 typewriter so that they will not have to be written again when further manipulation of the expression is desired.