STANFORD ARITIFICIAL INTELLIGENCE PROJECT July 20, 1964
Memo No. 17

FORMAL DESCRIPTION OF THE GAME OF PANG-KE

by John McCarthy

Abstract: The game of Pang-Ke is formulated in a first-order-logic in order to provide grist for the Advice-Taker mill. The memo does not explain all the terms used.

The research reported here was supported in part by the Advanced Research Project Agency of the Office of the Secretary of Defense (SD-183)

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The game of Pang-Ke is played on the board and with the initial position shown in figure 1. Players move alternately by moving a piece to a horizontally or vertically adjacent square. A piece is captured and removed from the board if an opponent moves into a line and causes the configuration |a|a|b| | or | |a|a|b|. To win one must capture all the opponent's pieces.

O	O	O	O
O			O
x			x
x	x	x	x

The following is a formulation of the game in first order logic. We use equality, functions and conditional expression in addition to predicate calculus.

Predicates

sq(s)	s is a square
n4(i)	i is a number between 1 and 4
i < j	i < j
pos (p)	p is a position
typ (u)	u is a possible occupant of a square
side (u)	u is a side (white or black)
won (u,p)	the side u has won in position p
initial (p)	p is the initial position
neighbor (s1,s2)	the squares s1 and s2 are neighbors.
ok move (s1,s2,p)	it is legal to move from square s1 to square s2 in position p
captured (s1,s2, s3,p)	s3 is captured in the move from s1 to s2 in position p.
legal(p1,p2)	it is legal to move from position p1 to p2

Constants

1, 2, 3, 4, black, white, blank

Functions

row(s) the number of the row in which square s is located
col(s) the number of the column in which square s is located
(i,j) the square in row i and column j
i' the successor of the number i
occ(s,p) the occupant of square s in position p
mover(p) the side whose turn it is in position p
opp(u) the opposite side to side u

Axioms

1. $\mathrm{sq}(s) \supset \mathrm{n4}(\mathrm{row}(s)) \wedge \mathrm{n4}(\mathrm{col}(s)) \wedge s = \gamma(\mathrm{row}(s), \mathrm{col}(s))$
2. $\mathrm{n4}(r) \wedge \mathrm{n4}(c) \supset \mathrm{sq}(\gamma(r,c)) \wedge r = \mathrm{row}(\gamma(r,c)) \wedge c = \mathrm{col}(\gamma(r,c))$
3. $\mathrm{n4}(i) \equiv i = 1 \vee i = 2 \vee i = 3 \vee i = 4$
4. $1 < 2 \wedge 2 < 3 \wedge 3 < 4$
5. $i < j \wedge j < k \supset i < k$
6. $\neg\, i < i$
7. $1' = 2 \wedge 2' = 3 \wedge 3' = 4$
8. $\mathrm{pos}(p) \wedge \mathrm{sq}(s) \supset \mathrm{typ}(\mathrm{occ}(s,p))$
9. $\mathrm{typ}(u) \supset u = \mathrm{black} \vee u = \mathrm{white} \vee u = \mathrm{blank}$
10. $\mathrm{pos}(p) \supset \mathrm{side}(\mathrm{mover}(p))$
11. $\mathrm{side}(u) \supset u = \mathrm{black} \vee u = \mathrm{white}$
12. $\mathrm{side}(u) \supset \mathrm{opp}(u) = \underline{\text{if}}\ u = \mathrm{black}\ \underline{\text{then}}\ \mathrm{white}\ \underline{\text{else}}\ \mathrm{black}$
13. $\mathrm{won}(p,u) \equiv \mathrm{pos}(p) \wedge \mathrm{side}(u) \wedge [\exists s.\ \mathrm{occ}(s,p) = u] \wedge \neg[\exists s.\ \mathrm{occ}(s,p) = \mathrm{opp}(u)]$
14. $\mathrm{initial}(p) \equiv \mathrm{mover}(p) = \mathrm{black} \wedge [\mathrm{occ}(p,\gamma(1,1)) = \mathrm{black} \wedge \mathrm{occ}(p,\gamma(1,2)) = \mathrm{black} \wedge \mathrm{occ}(p,\gamma(1,3)) = \mathrm{black} \wedge \mathrm{occ}(p,\gamma(1,4)) = \mathrm{black} \wedge \mathrm{occ}(p,\gamma(2,1)) = \mathrm{black} \wedge \mathrm{occ}(p,\gamma(2,2)) = \mathrm{blank} \wedge \mathrm{occ}(p,\gamma(2,3)) = \mathrm{blank} \wedge \mathrm{occ}(p,\gamma(2,4)) = \mathrm{black} \wedge \mathrm{occ}(p,\gamma(3,1)) = \mathrm{white} \wedge \mathrm{occ}(p,\gamma(3,2)) = \mathrm{blank} \wedge \mathrm{occ}(p,\gamma(3,3)) = \mathrm{blank} \wedge \mathrm{occ}(p,\gamma(3,4)) = \mathrm{white} \wedge \mathrm{occ}(p,\gamma(4,1)) = \mathrm{white} \wedge \mathrm{occ}(p,\gamma(4,2)) = \mathrm{white} \wedge \mathrm{occ}(p,\gamma(4,3)) = \mathrm{white} \wedge \mathrm{occ}(p,\gamma(4,4)) = \mathrm{white}$
15. $\mathrm{neighbor}(s1,s2) \equiv (\mathrm{row}(s1) = \mathrm{row}(s2)) \wedge [\mathrm{col}(s1)' = \mathrm{col}(s2) \vee \mathrm{col}(s2)' = \mathrm{col}(s1)] \vee (\mathrm{col}(s1) = \mathrm{col}(s2)) \wedge [\mathrm{row}(s1)' = \mathrm{row}(s2) \vee \mathrm{row}(s2)' = \mathrm{row}(s1)]$
16. $\mathrm{ok\ move}(s1,s2,p) \equiv \mathrm{neighbor}(s1,s2) \wedge \mathrm{occ}(s1,p) = \mathrm{mover}(p) \wedge \mathrm{occ}(s2,p) = \mathrm{blank}$
17. $\mathrm{captured}(s,s2,s3,p) \equiv \mathrm{ok\ move}(s,s2,p) \wedge \mathrm{occ}(s3,p) = \mathrm{opp}(\mathrm{mover}(p)) \wedge \exists s1 \exists s4 [s1 \neq s2 \wedge s1 \neq s3 \wedge s4 \neq s2 \wedge s4 \neq s3 \wedge s4 \neq s1 \wedge [\mathrm{row}(s1) = \mathrm{row}(s2) = \mathrm{row}(s3) = \mathrm{row}(s4) \vee \mathrm{col}(s1) = \mathrm{col}(s2) = \mathrm{col}(s3) = \mathrm{col}(s4)] \wedge \mathrm{occ}(s1,p) = \mathrm{mover}(p) \wedge (s = s4 \vee \mathrm{occ}(s4,p) = \mathrm{blank}) \wedge \mathrm{neighbor}(s1,s2) \wedge (\mathrm{neighbor}(s1,s3) \vee \mathrm{neighbor}(s2,s3))]$
18. $\mathrm{legal}(p1,p2) = \mathrm{mover}(p2) = \mathrm{opp}(\mathrm{mover}(p1)) \wedge \exists s1 \exists s2 [\mathrm{ok\ move}(s1,s2,p1) \wedge \forall s.\ \mathrm{sq}(s) \supset \mathrm{occ}(s,p2) = \underline{\text{if}}\ s = s1\ \underline{\text{then}}\ \mathrm{blank}\ \underline{\text{else if}}\ s = s2\ \underline{\text{then}}\ \mathrm{mover}(p1)\ \underline{\text{else}}\ \underline{\text{if}}\ \mathrm{captured}(s1,s2,s,p1)\ \underline{\text{then}}\ \mathrm{blank}\ \underline{\text{else}}\ \mathrm{occ}(s,p1)]$
19. $\mathrm{canwin}(u,p) \equiv \mathrm{won}(u,p) \vee [\neg\, \mathrm{won}(\mathrm{opp}(u,p)) \wedge \exists p1.\ \mathrm{legal}(p,p1) \wedge \underline{\text{if}}\ \mathrm{mover}(p) = u\ \underline{\text{then}}\ [\exists p1.\ \mathrm{legal}(p,p1) \wedge \mathrm{canwin}(u,p1)]\ \text{else}\ \forall p1.\ \mathrm{legal}(p,p1) \supset \mathrm{canwin}(u,p1)]$

This equation is satisfied by the condition that u can win in position p but it does not characterize it since there are predicates that satisfy the equation that show u winning some drawn situations.