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AESTHETICS SYSTEMS

BY

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COMPUTER SCIENCE DEPARTMENT  
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James Gips\*

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ABSTRACT: The formal structure of aesthetics systems is defined. Aesthetics systems provide for the essential tasks of interpretation and evaluation in aesthetic analysis. Kolmogorov's formulation of information theory is applicable. An aesthetics system for a class of non-representational, geometric paintings and its application to three actual paintings is described in the Appendix.

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A possible abstract structure for theories of aesthetics is defined. Aesthetics systems provide a logical framework in which the essential tasks of interpretation and evaluation are meaningful and possible. The formal organization of aesthetics systems is constant even though the content of specific systems may vary widely within and across such disciplines as painting, sculpture, literature, music, mathematics, and science.

An aesthetics system is given by the 4-tuple  $\langle I_A, R, E, O \rangle$ , where  $I_A$  is a set of interpretations defined by an algorithm  $A$ ,  $R$  is a reference decision algorithm which determines if an element of  $I_A$  refers to a given object,  $E$  is an evaluation function defined on  $I_A$ , and  $O$  is an order in the range of  $E$ . In an aesthetics system  $\langle I_A, R, E, O \rangle$ , the initial two components are called an interpretative system, the final two components an evaluative system.

$I_A$  contains all possible input-output pairs  $\langle \alpha, \beta \rangle$  for a fixed algorithm  $A$ , i.e., given finite input  $\alpha$ ,  $A$  terminates with finite output  $\beta$ , where both  $\alpha$  and  $\beta$  are non-empty strings over possibly different finite alphabets. Let  $e$  be the empty string, then

$$I_A = \{ \langle \alpha, \beta \rangle \mid A(\alpha) = \beta \wedge \alpha \neq e \wedge \beta \neq e \}$$

In aesthetics systems,  $\langle \alpha, \beta \rangle$  is called an interpretation. If  $A$  is a universal computing algorithm then  $\alpha$  is an encoding of a program and initial conditions which generates the sequence  $\beta$  when processed by  $A$ .

The reference decision algorithm  $R$  when presented with an interpretation in  $I_A$  and a real-world object decides whether the interpretation refers to the object.  $R$  contains a suitable sensory input transducer which

provides an interface with the object. The output of R is True if the interpretation refers to the object and False otherwise. In general, there are many interpretations in IA which do not refer to actual objects. Where the consideration of unformalized objects and sensory input transducers is unusual in formal systems, aesthetics systems would have no empirical significance without R. A discussion of reference in the context of aesthetics is given in (Goodman 1968).

Interpretative systems using the reference decision algorithm schema shown in Figure 1 are apposite to aesthetic analysis. Only one component of an interpretation is used as input in this schema. If the input is  $\alpha$  then the reference decision algorithm is denoted  $R_\alpha$ , if  $\beta$  then  $R_\beta$ . S, the first part of the schema, shows a sensory input transducer linked to an algorithm which produces a finite, discrete description or representation, i.e., formalization,  $\lambda$  of the presented object. For example, in music, drama, literature, architecture, or science A could resemble the score, text, plan, or data. The second part is a comparator which has as output True if  $\lambda$  is identical to the input component of the interpretation and False otherwise. If the input is  $\alpha$  then reference is decided exclusively in terms of  $\alpha$ , if  $\beta$  then exclusively in terms of  $\beta$ . In interpretative systems using  $R_\alpha$  or  $R_\beta$ ,  $\lambda$  is the complete description of the object in the sense that only those attributes identified by  $\lambda$  are considered in interpretations. Because  $\lambda$  is used as the complete description of the object, different objects producing identical  $\lambda$  are indistinguishable for interpretation. This allows a single interpretation to refer to multiple reproductions of a painting,

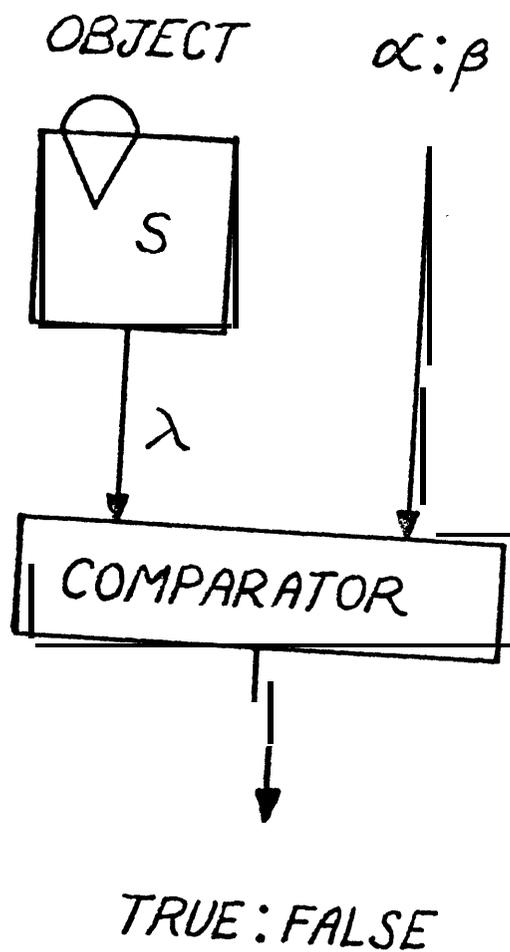


Figure 1. The reference decision algorithm schema for  $R_\alpha$  or  $R_\beta$ .

copies of a novel, performances of a concerto, or occurrences of a phenomenon.

Interpretative systems using  $R_\alpha$  may be considered to deal with the external evocations of objects referenced by interpretations in  $I_A$ . In these systems,  $\alpha$  occurring in an interpretation which refers to an object is identical to  $A$  and is the description of the object.  $\beta$  is the sequence of symbols which is produced when this description is processed by  $A$ . When  $R_\alpha$  is used, each object can have at most one interpretation which refers to it. Intuitively,  $\beta$  is a list of the "associations" or a statement of the "emotions" evoked by the description given by  $\alpha$ . In the arts, interpretation examining the external evocations of objects is discussed often in terms of representation (Gombrich 1960, 1963a) and expression (Gombrich 1963b).

Interpretative systems using  $R_\beta$  may be considered to deal with the internal coherence of objects referenced by interpretations in  $IA'$ . In these systems,  $\beta$  occurring in an interpretation which refers to an object is identical to  $\lambda$  and is the description of the object.  $\alpha$  is a sequence of symbols which when processed by  $A$  produces exactly this description. When  $R_\beta$  is used, an object can have more than one interpretation which refers to it, i.e., all interpretations which refer to a common object contain identical  $\beta$  but different  $\alpha$ . Intuitively,  $\alpha$  encodes the description given by  $\beta$  in terms of the syntactic or semantic redundancy, organization, or pattern underlying it. In the arts, interpretation examining the internal coherence of objects is discussed often in terms of composition and form (Focillon 1948). In science, the logical structure of the phenomenon :: data :: theory paradigm is an instance of the object ::  $\beta$  :: a relation

in interpretative systems using  $R_\beta$ . The sensory input transducer and linked algorithm of  $R_\beta$  correspond to the data collection mechanism  $A$  corresponds to the mathematical conventions implicit in the theory. Just as in science the theory provides an encoding for the data describing the phenomenon,  $A$  provides an encoding for the description  $\beta$  of the object.

Interpretative systems  $\langle I_{A_1}, R_{\alpha_1} \rangle$  and  $\langle I_{A_2}, R_{\beta_2} \rangle$  can be combined to form a single interpretative system  $\langle I_{A_3}, R_{\alpha_3} \rangle$  if  $R_{\alpha_1}$  is identical to  $R_{\beta_2}$ , i.e.,  $\lambda = \alpha_1 = \beta_2$  for the same object. The reference decision algorithm  $R_3$  can be constructed effectively using either  $A_1$  or  $A_2$  and the common reference decision algorithm. The algorithm  $A_3$  associated with the set of interpretations  $I_{A_3}$  is the composition of  $A_2$  and  $A_1$ . Interpretations in  $I_{A_3}$  have the form  $\langle \alpha_2, \beta_1 \rangle$ , where  $\lambda$  is given internally in  $A_3$  and  $R_3$ . Intuitively, this new interpretative system may be considered to deal with the relationship between the internal coherence and external evocations of objects referenced by interpretations in  $I_{A_3}$ .

The distinction between external evocation and internal coherence and their relationship is discussed by Beardsley (1958) in terms of "critical interpretation" and "critical description." An interpretative system embodies a particular interpretative viewpoint. All interpretations consistent with the underlying assumptions of the viewpoint are elements of  $I_A$ .  $I_A$  defines the potential scope of an interpretative viewpoint;  $R$  determines its empirical extent. Any interpretative viewpoint is allowable if  $A$  and  $R$  can be constructed to conform to its conventions. This possibility of varied content within an invariant formal system can account for the relativity of aesthetic experience.

The evaluation function  $E$  is defined on the set  $IA$ .  $0$  is an order defined in the range of  $E$  and may be partial or total. The evaluation function together with the order ranks elements of  $IA$ .

An appropriate evaluation function for aesthetics systems is given by

$$E_Z(\langle \alpha, \beta \rangle) = \frac{L(\beta)}{L(\alpha)}$$

where  $L(a)$  is the length of  $\alpha$  and  $L(B)$  is the length of  $\beta$ . Following Kolmogorov (1968), if  $\alpha$  is defined over a binary alphabet and if  $\alpha$  is the shortest string such that  $A(\alpha) = \beta$  then  $L(a)$  is the information-theoretic complexity or entropy of  $\beta$  with respect to  $A$ . The total order  $0_Z$  naturally associated with  $E_Z$  would rank two interpretations such that the interpretation assigned the higher value is aesthetically superior.

The evaluative system  $\langle E_Z, 0_Z \rangle$  can be combined with any interpretative system to form an aesthetics system. For a given  $IA$ , the evaluation function  $E_Z$  assigns high aesthetic values to interpretations containing  $\beta$  which have an identifiably redundant, periodic, or regular structure with respect to  $A$  as encoded by  $\alpha$ . In this case,  $\alpha$  is an economical specification of  $\beta$  using  $A$ . Interpretations which contain  $\beta$  which are random (Kolmogorov 1968) with respect to  $A$  are assigned lowest values. In this case, there is no  $\alpha$  which is an economical specification of  $\beta$  using  $A$ . For a description  $\lambda$  of an object, an interpretation in an interpretative system using  $R_\alpha$  is assigned a relatively high aesthetic value when  $\lambda$  has multiple evocations, in an interpretative system using  $R_\beta$  when  $\lambda$  has a simple encoding. For interpretations which refer to objects, the length of  $A$  usually is limited by the acuity of  $S$ . For interpretations (in  $R_\beta$  systems) which refer to the same

object, the use of  $\langle E_Z, O_Z \rangle$  produces a ranking of these interpretations which corresponds to the application of Occam's razor.

The evaluative system  $\langle E_Z, O_Z \rangle$  follows a long tradition of aesthetic evaluation. In the arts, Fechner's discussion of "unity" and "variety" (Fechner 1897) and Birkhoff's investigation of "order" and "complexity" (Birkhoff 1932), (Eysenck 1941) are analogous to the combination of  $\langle E_Z, O_Z \rangle$  with interpretative systems using  $R_\beta$ . In this context, "unity" and "order" may be associated with  $L(\alpha)$ , "variety" and "complexity" with  $L(\beta)$ , i. e.,  $L(\lambda)$ . Beardsley's discussion of "unity," "complexity," and "intensity" (Beardsley 1958) is analogous to the combination of  $\langle E_Z, O_Z \rangle$  with interpretative systems formed by the composition of interpretative systems using identical  $R_\alpha$  and  $R_\beta$ . In this context, "unity" may be associated with  $L(a)$ , "complexity" with  $L(A)$ , and "intensity" with  $L(B)$ . In science, interpretative systems using  $R_\beta$  are employed and evaluation is considered frequently in terms of the law of parsimony or Occam's razor, cf. (Rossi 1956). The everyday use of the words "beautiful" and "elegant" to describe mathematical systems and physical laws is in the spirit of  $\langle E_Z, O_Z \rangle$ --parsimonious specification of complicated phenomena.

The structure of aesthetics systems has been described independent of the content of any specific system. Two examples of aesthetics systems, one in art and one in science, are discussed briefly.

An aesthetics system for non-representational, geometric paintings has been constructed (see Appendix).  $\alpha$  is given in terms of generative specifications (Stiny and Gips 1972), which are based on shape grammars.  $\beta$  is given in terms of shape, color, and occurrence tables where there is

a one-to-one correspondence between each entry in the occurrence table and each distinct area occurring in the painting referenced by the interpretation. The  $R_{\beta}$  reference decision algorithm schema and the evaluative system  $\langle E_Z, O_Z \rangle$  are used. Computer implementation of this aesthetics system is in progress.

The Meta-Dendral system (Buchanan, Feigenbaum, and Lederberg 1971), a program for automatic theory formation in mass-spectrometry, embodies implicitly an aesthetics system  $\alpha$  is-given in terms of situation-action rules constituting an hypothesized subset of the theory of mass-spectrometry and a list of molecular structures.  $\beta$  is given in terms of the fragment mass tables for each molecular structure given in  $\alpha$ . The  $R_{\beta}$  schema and an evaluative system similar to  $\langle E_Z, O_Z \rangle$  are used. Because the molecular structure - fragment mass table pairs are held constant for all interpretations, the interpretation that is assigned the highest aesthetic value contains minimal situation-action rules.

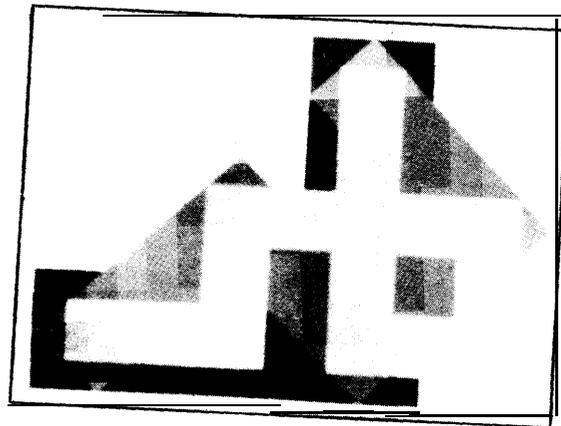
Aesthetics systems are useful in the investigation of a wide variety of traditional problems in art theory and criticism, including design and style. Design can be formulated in terms of heuristic search of a structured space of interpretations defined by a specific aesthetics system. The goal of this search is the identification of interpretations having high aesthetic values. Art objects with interpretations having high aesthetic values in a given aesthetics system can be said to be in the same style. A discussion of these issues and a more detailed analysis of the role of aesthetics systems in art theory is given in (S-tiny and Gips in preparation).

**Acknowledgement:** We wish to thank Tom Cover of Stanford for extending our familiarity with the current literature of information theory.

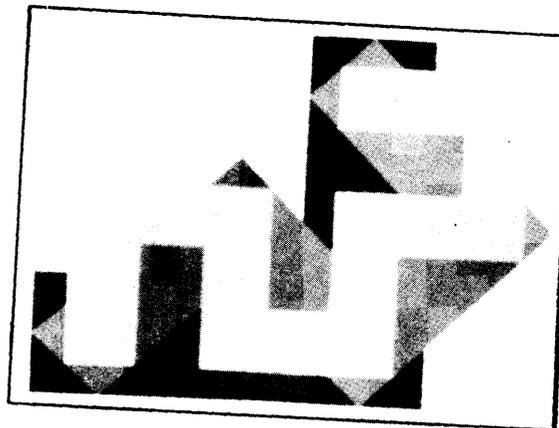
## APPENDIX

In this section we outline an aesthetics system which contains interpretations which refer to paintings specifiable by generative specifications (Stiny and Gips 1972). Computer implementation of this aesthetics system is in progress. The three paintings, Anamorphism I, II, and III, shown in Figure 2 are used as an example. Interpretations which refer to these paintings are given and these interpretations are ranked.

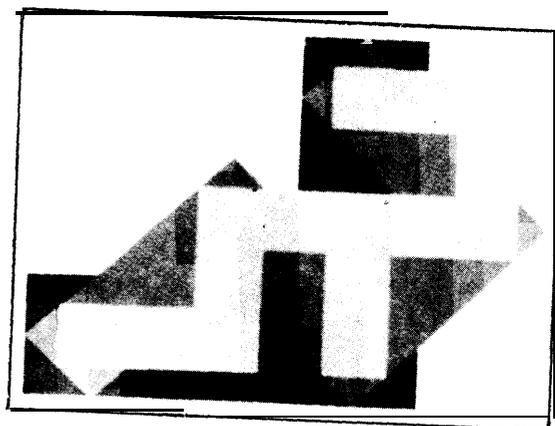
An interpretation has the form  $\langle \alpha, \beta \rangle$ . In this aesthetics system  $\alpha$  is given by a generative specification. Briefly, a generative specification consists of a shape specification, which determines a class of shapes, and a material specification, which determines how these shapes are represented materially. A shape specification consists of a shape grammar and a selection rule. A shape grammar is similar to a phrase structure grammar. Where a phrase structure grammar is defined over an alphabet of symbols and generates one-dimensional strings of symbols, a shape grammar is defined over an alphabet of shapes and generates n-dimensional shapes. A selection rule selects shapes from the language of shapes defined by a shape grammar and provides a halting algorithm for the shape generation process. A material specification consists of a finite list of painting rules and a limiting shape. Painting rules indicate how the areas contained in a shape are colored by considering the shape as a Venn diagram as in naive set theory. The limiting shape has the properties of a camera viewfinder, determining what part of a painted shape occurs on a canvas of given size and shape and in what orientation and scale. Figure 3 shows the generative specification



Anamorphism I



Anamorphism II



Anamorphism III

Figure 2. Anamorphism I, II, and III.  
Legend: darkest - blue; second darkest - red;  
lightest - yellow; lightest - light blue.

# GENERATIVE SPECIFICATION

## SHAPE SPECIFICATION

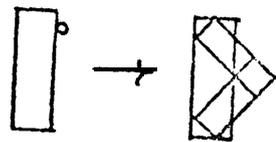
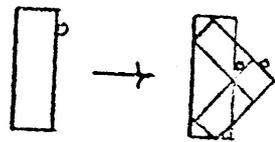
## MATERIAL SPECIFICATION

### SHAPE GRAMMAR

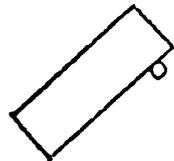
$$V_T = \{ \square \}$$

$$V_M = \{ \circ \}$$

Rules:



Initial Shape:



SELECTION RULE

$$\langle 3, 3 \rangle$$

### PAINTING RULES

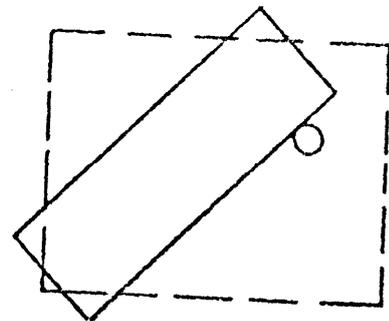
$$L3 \Rightarrow \text{YELLOW}$$

$$L2 \cap \sim L3 \Rightarrow \text{RED}$$

$$L1 \cap \sim (L2 \cup L3) \Rightarrow \text{BLUE}$$

$$\sim (L1 \cup L2 \cup L3) \Rightarrow \text{LIGHT BLUE}$$

LIMITING SHAPE



20 ms.

Figure 3a.  $\alpha$  for Anamorphism I.

# GENERATIVE SPECIFICATION

SHAPE SPECIFICATION

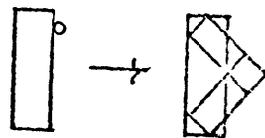
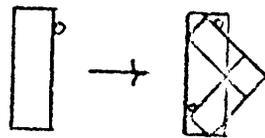
MATERIAL SPECIFICATION

## SHAPE GRAMMAR

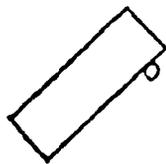
$$V_T = \{ \square \}$$

$$V_M = \{ \circ \}$$

Rules:



Initial Shape:



SELECTION RULE

$\langle 3, 3 \rangle$

## PAINTING RULES

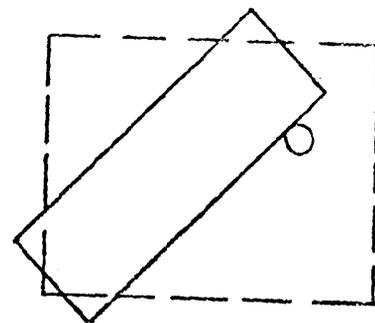
$L_3 \Rightarrow$  YELLOW

$L_2 \cap \sim L_3 \Rightarrow$  RED

$L_1 \cap \sim (L_2 \cup L_3) \Rightarrow$  BLUE

$\sim (L_1 \cup L_2 \cup L_3) \Rightarrow$  LIGHT BLUE

## LIMITING SHAPE



20 ins.

Figure 3b.  $\alpha$  for Anamorphism II.

# GENERATIVE SPECIFICATION

## SHAPE SPECIFICATION

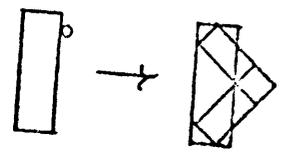
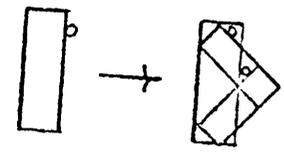
## MATERIAL SPECIFICATION

### SHAPE GRAMMAR

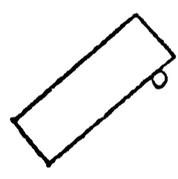
$$V_T = \{ \square \}$$

$$V_M = \{ \circ \}$$

Rules:



Initial Shape:



SELECTION RULE

$$\langle 3, 3 \rangle$$

### PAINTING RULES

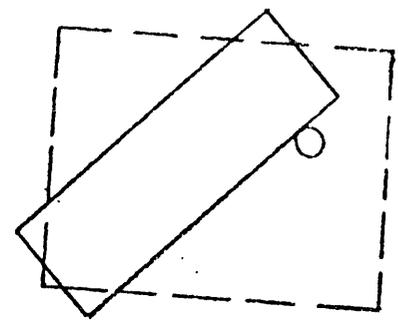
$$L_3 \Rightarrow \text{YELLOW}$$

$$L_2 \cap \sim L_3 \Rightarrow \text{RED}$$

$$L_1 \cap \sim (L_2 \cup L_3) \Rightarrow \text{BLUE}$$

$$\sim (L_1 \cup L_2 \cup L_3) \Rightarrow \text{LIGHT BLUE}$$

### LIMITING SHAPE



20 ins.

Figure 3c.  $\alpha$  for Anamorphism III.

for Anamorphism I, II, and III. Note that the specifications differ only in the placement of the markers, i.e., circles, in the right side of the first rule in the shape grammars.

$\beta$  occurring in interpretations in this aesthetics system consist of three tables which have the general format indicated in Figure 4. Each entry in the occurrence table corresponds uniquely to a distinct colored area occurring in a painting. Each entry has seven parts:  $i_s$  is the index of a shape occurring in the shape table and specifies the shape of the area;  $i_c$  is the index of a color occurring in the color table and specifies the color of the area;  $x$ ,  $y$ ,  $\theta$ ,  $s$ , and  $m$  are transformations which map the shape indexed by  $i_s$  from the shape table coordinate system to the painting coordinate system, where  $x$  and  $y$  determine translation,  $\theta$  determines rotation,  $s$  determines scale, and  $m$  determines if the mirror image of the shape is used. Entries in the shape table correspond to the different shapes of the areas occurring in a painting. Entries in the color table correspond to the different colors of the areas occurring in a painting. For Anamorphism I, II, and III, each occurrence table has twenty entries as there are twenty distinct colored areas in each painting. Each color table has four entries as there are four different colors in each painting. For Anamorphism I the shape table has seven entries as there are seven different shapes occurring in the painting. For Anamorphism II, the shape table has six entries, for Anamorphism III five entries (see Figure 5).

For both  $\alpha$  and  $\beta$ , the computer representation of closed, rectilinear shapes is constructed by fixing two of the vertices of the shape and listing the  $(x,y)$  coordinates of the remaining vertices in the order of a

### SHAPE TABLE

$i_s$	Shape
1	
2	
⋮	⋮
⋮	⋮

### OCCURRENCE TABLE

$i_s$	$i_c$	$x$	$y$	$\theta$	$s$	$m$
$i_{s1}$	$i_{c1}$	$x_1$	$y_1$	$\theta_1$	$s_1$	$m_1$
$i_{s2}$	$i_{c2}$	$x_2$	$y_2$	$\theta_2$	$s_2$	$m_2$
			⋮			
			⋮			

### COLOR TABLE

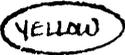
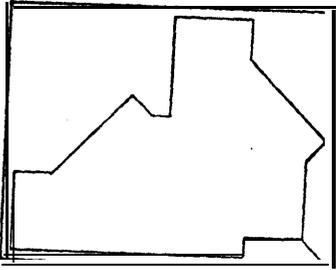
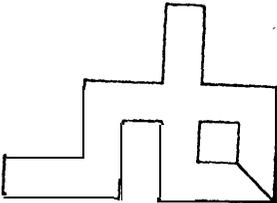
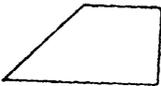
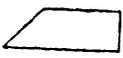
$i_c$	Color
1	
2	
⋮	⋮
⋮	⋮

Figure 4. Table format for  $\beta$ .

$i_s$	Shape		Length 2V-3
1		20	37
2		20	37
3		5	7
4		4	5
5		4	5
6		4	5
7		3	3

99

Figure 5a. Length of Shape Table of  $\beta$  for Anamorphism I.

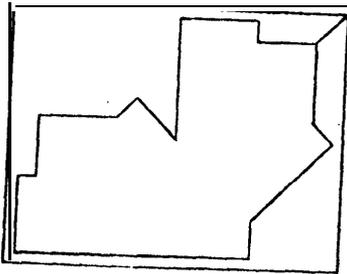
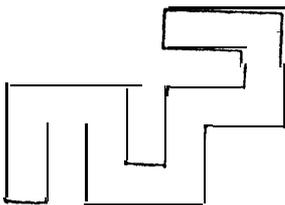
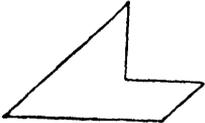
$i_s$	Shape	$v$	Length $2v-3$
1		21	39
2		18	33
3		5	7
4		5	7
5		4	5
6		3	3
			94

Figure 5b. Length of Shape Table of  $\beta$  for Anamorphism II.

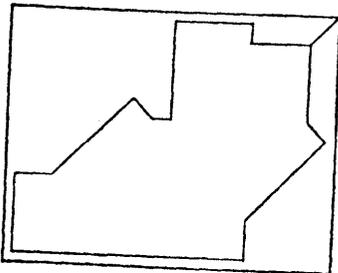
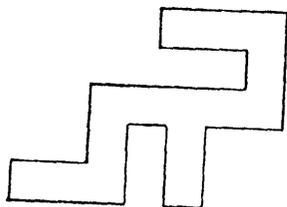
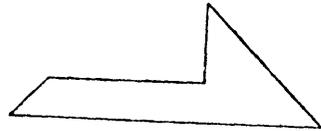
$i_s$	Shape	$V$	Length $2V-3$
1		20	37
2		16	29
3		5	7
5		4	5
		3	3
			81

Figure 5c. Length of Shape Table of  $\beta$  for Anamorphism III.

counter-clockwise trace around the boundary of the shape. Holes in shapes result in the construction of two identical edges between a vertex on the inner boundary and a vertex on the outer boundary so that the trace around the shape is continuous. Because the number of vertices of shapes may vary, the first entry in the representation is the number of coordinates listed. The number of words of memory used to represent each shape in this format is  $2(V-2)+1 = 2V-3$  where  $V$  is the number of vertices encountered in a complete trace around the shape. For both  $\alpha$  and  $\beta$ , the computer representation of color is given by three words of memory containing the intensities of the red, blue, and green components of the color.

$R$  in this aesthetics system can be constructed to correspond with the  $R_\beta$  schema. The sensory input transducer of  $S$  would be a color television camera; the algorithm of  $S$  would contain an edge following routine. The  $\lambda$  constructed by  $S$  would be equivalent to  $\beta$ .

In the calculation of aesthetic value using  $E_Z$  in this aesthetics system, the lengths of  $\alpha$  and  $\beta$  are equivalent to the number of words of computer memory used to encode them. In the given interpretations which refer to Anamorphism I, II, and III, the lengths of the computer representations of the generative specifications are equal because these specifications differ only in the positions of the markers in their respective shape grammars. The lengths of the computer representations of the occurrence tables are equal because the number of entries in each table is the same and the number of words of memory required for each entry is constant. Similarly for the color tables. Since  $L(a)$  is the same in each interpretation and  $L(B)$  differs only in the lengths of the computer representations

of the shape tables, the aesthetic values assigned to the given interpretations which refer to Anamorphism I, II, and III are directly proportional to the lengths of the computer representations of their respective shape tables. Figure 5 shows the shapes occurring in the shape tables and the computation of the lengths of the computer representations of these tables. In this aesthetics system the interpretation given for Anamorphism I has a higher aesthetic value than the interpretation given for Anamorphism II; the interpretation given for Anamorphism II has a higher aesthetic value than the interpretation given for Anamorphism III.

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